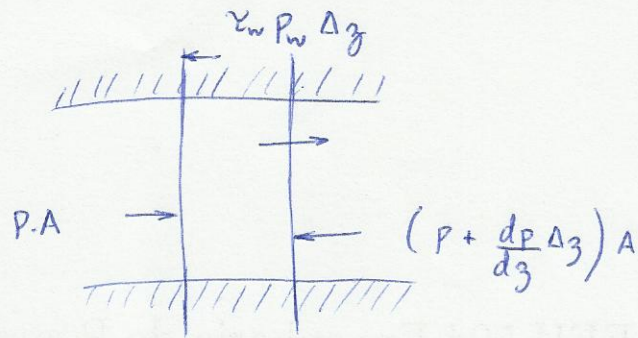


$$\tau_w = \left(-\frac{dp}{dz}\right)$$



$$P \cdot A - \left(P + \frac{dp}{dz} \Delta z\right) A - \tau_w P_w \Delta z = 0$$

$$\left(-\frac{dp}{dz}\right) \cancel{\Delta z} \cdot A = \tau_w P_w \cancel{\Delta z} \quad \left\{ \begin{array}{l} A = \pi \frac{D^2}{4} \\ P_w = \pi D \end{array} \right.$$

$$\left(-\frac{dp}{dz}\right) \cancel{\pi} \frac{D^2}{4} = \tau_w \cancel{\pi} D$$

$$\tau_w = \left(-\frac{dp}{dz}\right) \frac{D}{4}$$

$$\frac{U_\tau}{U_m} = \frac{1}{U_m} \sqrt{\frac{\tau_w}{\rho}} = \sqrt{\left(-\frac{dp}{dz}\right) \frac{D}{4 \rho U_m^2}} = \sqrt{\frac{1}{8} \left(-\frac{dp}{dz}\right) \frac{D}{\frac{1}{2} \rho U_m^2}} = \sqrt{\frac{f}{8}}$$

$$\frac{U_m}{U_\tau} = \sqrt{\frac{8}{f}}$$

Depois de integrar $u(r)$: $U_m = U_\tau \left(C_1 \ln \frac{R U_\tau}{\nu} + C_2 \right)$

$$\ln \frac{R U_\tau}{\nu} = \ln \frac{D}{2} \frac{U_m}{U_\tau} \cdot \frac{U_\tau}{U_m}$$

$$\ln \frac{Re}{2} \sqrt{\frac{f}{8}} = \ln \frac{R U_\tau}{\nu}$$

$$\frac{R U_\tau}{\nu} = \frac{Re}{2} \sqrt{\frac{f}{8}}$$